A derivation of Curve's Stable Swap price and slippage

Arthur Bagourd

May 2022

Abstract: *We follow on Curve's Stable Swap paper and derive the price and the slippage of a trade in a stable pool.*

For a pool with n stable coins, the StableSwap invariant writes:

$$A \cdot n^n \cdot \sum x_i + D = A \cdot D \cdot n^n + \frac{D^{n+1}}{n^n \cdot \prod x_i}$$

where:

- $D = \sum x_i$ is the constant-sum invariant such that the constant-product invariant is $\left(\frac{D}{n}\right)^n = \prod x_i$
- A is the amplification factor

We can rewrite the StableSwap invariant as a function f:

$$f(x_1, ..., x_n) = A \cdot n^n \cdot \sum x_i + D - A \cdot D \cdot n^n - \frac{D^{n+1}}{n^n \cdot \prod x_i}$$

such that

$$f(x_1, ..., x_n) = 0$$

The exchange rate of stable coin j in terms of stable coin k is given by $r_{j,k} = -\frac{dx_j}{dx_k} = \frac{\frac{df}{dx_k}}{\frac{df}{dx_j}}$ with:

$$\frac{df}{dx_j} = A \cdot n^n - \frac{D^{n+1}}{n^n \cdot \prod_{i \neq j} x_i} \cdot \left(-\frac{1}{x_j^2}\right)$$

$$= A \cdot n^n + \frac{D^{n+1}}{n^n \cdot x_j \cdot \prod x_i}$$

And we have:

$$r_{j,k} = \frac{x_j \cdot (x_k \cdot A \cdot n^{2n} \cdot \prod x_i + D^{n+1})}{x_k \cdot (x_j \cdot A \cdot n^{2n} \cdot \prod x_i + D^{n+1})}$$

Similarly, the slippage $s_{j,k}$ being the derivative of the price, we have

$$s_{j,k} = -\frac{dx_j}{dx_k^2} = \frac{\frac{df}{dx_k^2}}{\frac{df}{dx_j}}$$

with

$$\frac{df}{dx_k^2} = \frac{A \cdot n^n + \frac{D^{n+1}}{n^n \cdot x_k \cdot \prod x_i}}{dx_k}$$
$$= -\frac{2 \cdot D^{n+1}}{x_k^2 \cdot n^n \cdot \prod x_i}$$

And we have

$$s_{j,k} = \frac{-\frac{2 \cdot D^{n+1}}{x_k^2 \cdot n^n \cdot \prod x_i}}{A \cdot n^n + \frac{D^{n+1}}{n^n \cdot x_j \cdot \prod x_i}}$$
$$= -\frac{2 \cdot x_j \cdot D^{n+1}}{x_k^2 \cdot (A \cdot \prod x_i \cdot n^{2n} \cdot x_j + D^{n+1})}$$