On Hedging Validation Rewards

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Abstract: Whether an institutional staker or a humble node running shop, you receive rewards in ETH several times an hour*. On a longer time scale (week/month), you can pretty accurately estimate the number of ETH you will receive per node for the coming weeks/months. Now, your costs of running the nodes (the infra, the team, etc) is in USD (or any other fiat for that matter). This means that if ETH is crashing down while your costs remain the same - as you may have experienced in 2022 - you are making way less revenue and are potentially on the road to bankruptcy**. And, unlike Bitcoin, you cannot just stop your nodes and get the ETH back because withdrawals are not enabled (pre-merge) or because there is a big queue (post-merge).

As in the commodity world, it is good practice to hedge oneself against price variations. This paper details two solutions to do so.

1 A concrete example

Let's say you are running 1,000 nodes. You estimate that, on average, each node will earn 0.1 ETH per month, yielding a revenue of 100 ETH per month. At the end of each month, you can sell your 100 ETH at spot and get fiat to pay your running costs.

Let's say the ETH price at the start of the month is $2,800 as in Jan. 2022, if you were to sell your ETH at the end of the month, you would sell them at $2,000, i.e. $\sim$ 30% lower than at the start of the month. And at the end of June 2022 at $879 resulting in a profit of $ 87,900 vs $280,000 at the beginning of the year, resulting in a fat 68 % decline in revenue.

If you could agree a long time in advance to sell your ETH forward to a counterparty at a predetermined price you would avoid this 68% decline in revenue. For example, you could agree with the counterparty to sell 100 ETH at the end of each month at a price of $2,800 for the next 12 months. This is

*Actually if you have k validators and the network is made of n validators, you receive rewards every $6.4 \cdot \frac{n}{k}$ minutes on average as rewards are paid to the network every epoch and an epoch consists of 32 slots of 12s. In reality, this will be a bit different as the probability of being chosen also depends on the ETH balance of the node.

** If you are not saved in-extremis by VCs, raising at a despicable valuation.

called entering into a future agreement. The issue is that if the price goes up, you don't sell at a higher price, so you miss on potential gains.

In that case, you could hedge with a Put option. For each of the coming months, you buy a Put option on 100 ETH with a strike price of $2,800. So the less you can sell your ETH at is $2,800 and if the spot price is higher, you sell your ETH at the higher price. This way, you do not miss on the potential upside.

With a regular stream of income from validators, another way to hedge yourself is to swap your stream of variable rewards against a fixed stream of income. This way you pay a floating/uncertain value and receive a fixed payment. It is effectively like rolling a future contract over time.

2 Hedging with futures

We define:

- n the monthly number of ETH produced by the node runner
- S_t the spot price at time t
- $F_{t,T}$ the future price at time t with expiry at T
- r, the funding rate

We have:

$$F_{t,T} = S_t \cdot e^{-r \cdot (T-t)}$$

Such that if you expect to receive n ETH at the end of the month, you short n futures (assuming the size of a future is 1 ETH) and receive $n \cdot F_{t,T}$. At the end of the month you deliver your n ETH to the buyer of the future. In practice, a lot of futures are cash-settled, meaning that at the end of the month, you will sell your n ETH at spot S_T and buy back your future at $F_{T,T} = S_T$. The payoff is given by:

$$\underbrace{n \cdot F_{t,T}}_{\text{short future that you hold until physical settlement}}$$

or if you settle in cash:

$$= \underbrace{n \cdot F_{t,T}}_{\text{short future}} + \underbrace{n \cdot S_T}_{\text{rewards selling}} - \underbrace{n \cdot F_{T,T}}_{\text{future buyback}}$$

as $F_{T,T} = S_T$.

If $S_T > F_{t,T}$ then you will have missed on the upside but if $S_T < F_{t,T}$ then you are hedged against the loss. Potentially missing on the upside can be seen as problematic but it is the cost to pay for certainty.

3 Hedging with Swaps

We want to "swap" the floating income stream of validator rewards for a fixed income stream. The NPV of the flow of rewards over the incoming 12 months can be computed as such:

$$\sum_{i=1}^{12} \frac{n_i}{(1+r)^i}$$

where:

- n_i is the expected reward in ETH for month i
- r is the risk-free rate i is the month $i \in \{1, 2, ..., 12\}$

Now, at the end of each month i, the validator will receive K a fixed payment in exchange for n_i. K is such that:

$$\sum_{i=1}^{12} \frac{K}{(1+r)^i} = \sum_{i=1}^{12} \frac{\mathrm{E}[n_i]}{(1+r)^i}$$

or

$$K \cdot \sum_{i=1}^{12} \frac{1}{(1+r)^i} = \sum_{i=1}^{12} \frac{\mathrm{E}[n_i]}{(1+r)^i}$$

i.e.

$$K = \frac{\sum_{i=1}^{12} \frac{\mathrm{E}[n_i]}{(1+r)^i}}{\sum_{i=1}^{12} \frac{1}{(1+r)^i}}$$

In the above equations, K is in ETH, and what the validator needs is USD, so really, if we define K as a fixed payment in USD, we have:

$$K = \frac{\sum_{i=1}^{12} \frac{\mathrm{E}[n_i \cdot p_i]}{(1+r)^i}}{\sum_{i=1}^{12} \frac{1}{(1+r)^i}}$$

where p_i is the ETHUSD price during month i.

Are n_i and p_i independent? On a long enough time horizon they are not, as prices go up, the number of validators should increase and the reward per validator decrease. But it takes time to spin up the required infrastructure, meaning that on a monthly horizon n_i and p_i are independent. This implies that we can rewrite $\mathrm{E}[n_i \cdot p_i]$ as $\mathrm{E}[n_i] \cdot \mathrm{E}[p_i]$.

The best estimate of p_i is $\mathrm{E}[p_i] = F_{t,i}$. So we can rewrite K as:

$$K = \frac{\sum_{i=1}^{12} \frac{\mathrm{E}[n_i] \cdot F_{t,i}}{(1+r)^i}}{\sum_{i=1}^{12} \frac{1}{(1+r)^i}}$$

4 Hedging with Put options

At time t the price of a Put with expiry T and strike K is given by:

$$P(t,T,K) = K \cdot e^{-r\cdot(T-t)} \cdot N(-d_2) - S_t \cdot N(-d_1)$$

with:

- $d_1 = \frac{\ln(\frac{S_t}{K}) + \left(r + \frac{\sigma^2}{2}\right)\cdot(T-t)}{\sigma\cdot\sqrt{T-t}}$
- $d_2 = d_1 - \sigma \cdot \sqrt{T-t}$
- σ the volatility of ETH returns
- r the risk-free rate
- N is the normal standard CDF

The payoff of a Put is given by:

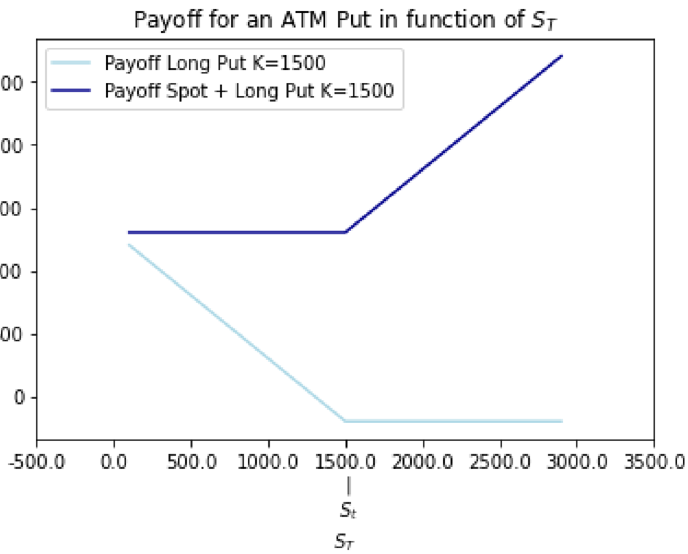
$$\max(0, K - S_T)$$

At T, if $S_T > K$, then the validator can sell your ETH for S_T and you effectively make $n \cdot (S_T - P(t,T,K)$, i.e. you have to bear the insurance cost of buying the put options. But if $S_T < K$ then you can sell your ETH at K. The PnL is given by:

$$\begin{aligned} PnL &= n \cdot (\mathbb{1}\{S_T < K\}K + \mathbb{1}\{S_T \geq K\}S_T - P(t,T,K)) \\ &= n \cdot (\max(S_T, K) - P(t,T,K)) \end{aligned}$$

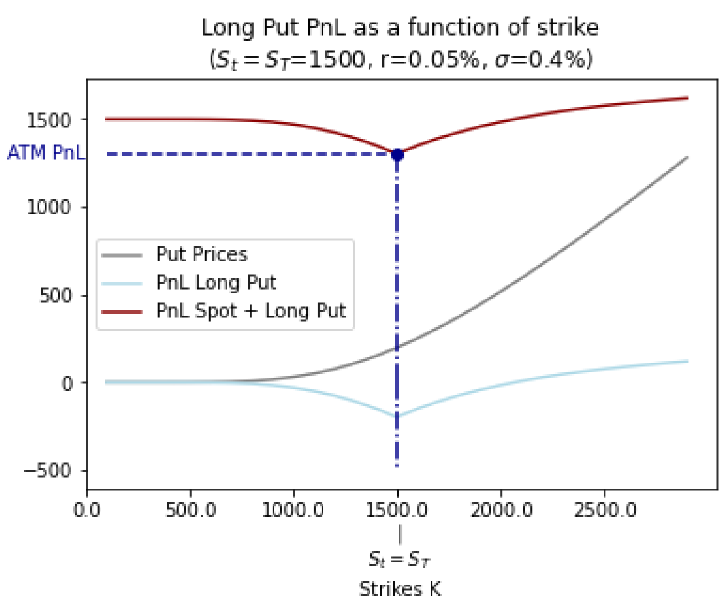
Assuming the size of a put option is 1 ETH, you would need to buy n monthly put options every month, or $3n$ 3M options every 3 months, which is less expensive but requires you to wait until maturity to see where the price is, so if the asset's price rebounds before maturity you could not be able to exercise the option.

Assuming a volatility of 40% and if we are long an ATM put the PnL as a function of spot price is given by:

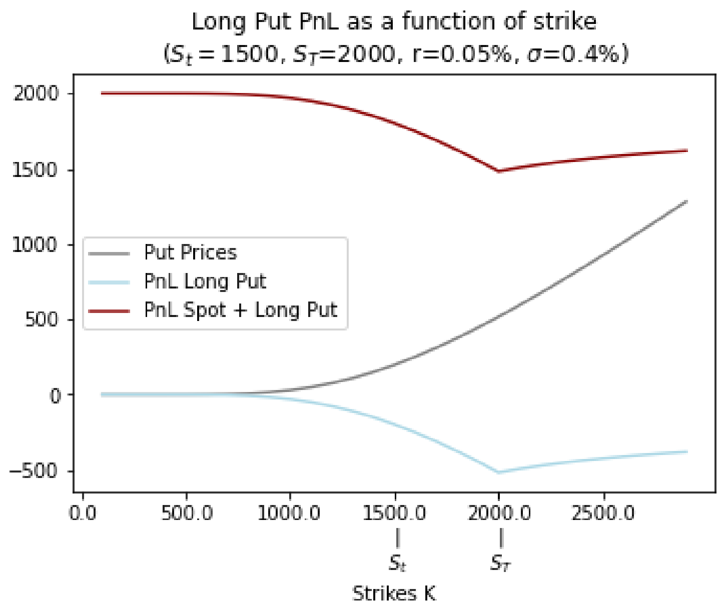


4.1 Put price as a function of strike

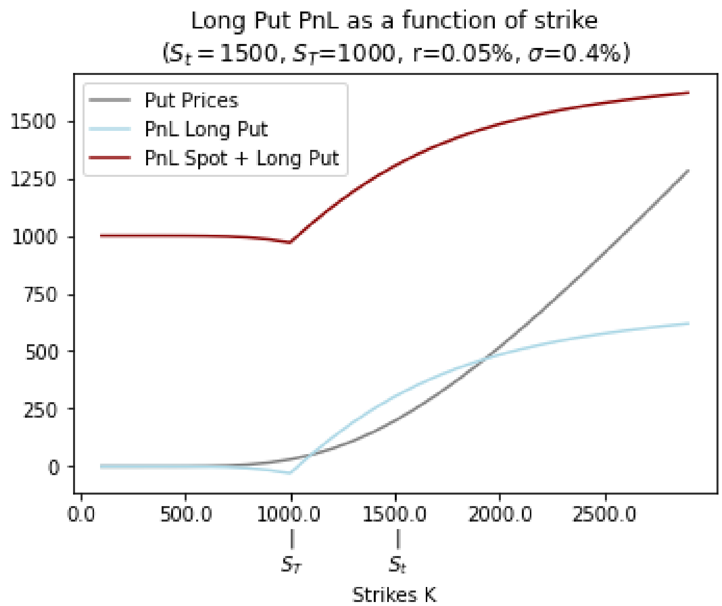
Now, how does the PnL varies with the chosen strike? Assuming the spot price is the same at maturity, the PnL as a function of strike is given by:



If we assume spot price rose from $S_t = 1,500$ to $S_T = 2,000$ the PnL is now given by:



If we assume spot price decreased from $S_t = 1,500$ to $S_T = 1,000$ the PnL is now given by:



In the end, the strike choice is dependent on the staker's costs constraints, and PnL optimization.

4.2 Zero Cost Collar

The issue with single put options is that the premium might be high, especially for not so much OTM puts, and because there is not much liquidity yet (not like in FX or Equities), market makers may add a hefty premium on top.

To make it cheaper, one can enter into a collar. It involves buying the put and selling a call at a higher strike, so that you cover the costs of your put but you forfeit the upside past a certain price. We give below the formula for the Call price:

$$C(t,T,K) = S \cdot N(d_1) - K \cdot \cdot e^{-r\cdot(T-t)} \cdot N(D_2)$$

As the PnL of a collar is given by:

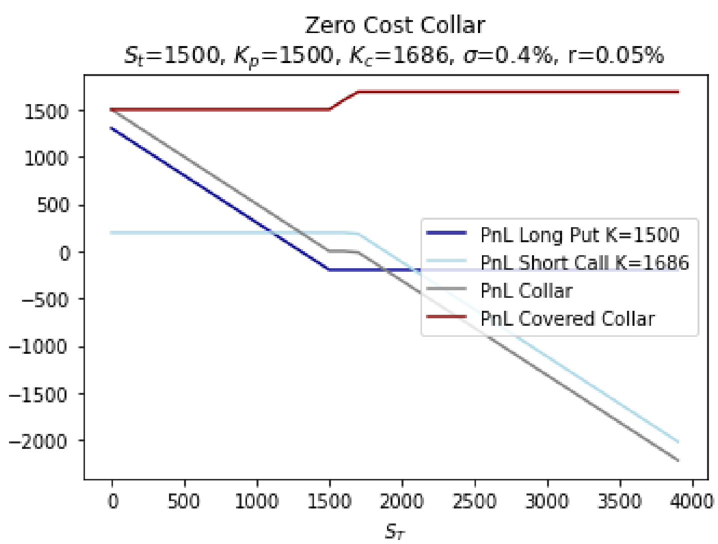
$$PnL = \max(0, K_p - S_T) - P(t,T,K_p) + C(t,T,K_c) - \max(0, S_T - K_c)$$

with

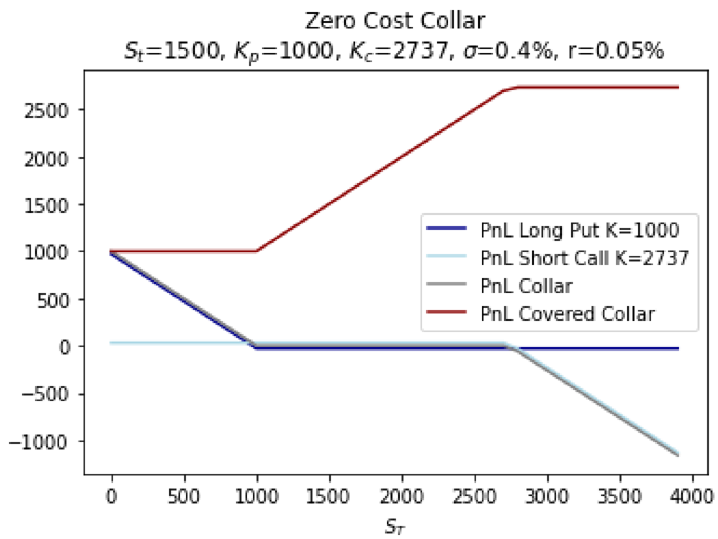
- K_p the chosen strike for the put
- K_c the chosen strike for the call

We can solve for K_c such that $P(t,T,K_p) = C(t,T,K_c)$, i.e. selling the call is covering the cost of buying the put.

The PnL of a zero-cost collar with an ATM put as a function of the final spot price is given by:



The PnL of a zero-cost collar with an OTM put as a function of the final spot price is given by:



We can see that by buying a more OTM and thus cheaper put, this allows us to sell a more OTM call to cover the cost, and hence we receive more of the upside of the underlying before forfeiting it.

4.3 Keeping the long tail upside performance of the underlying

What if ETH goes to the moon? If ETH price goes to \$10,000 and you are hedged with a zero cost collar with an ATM put and a call strike at \$1,600 then you will never get more than \$1,600 and you are missing on a huge upside. This can be overcome by buying a deep OTM call at \$3,000 for example. So that if the price goes to \$5,000 you can buy the underlying for \$3,000 and sell it for \$10,000.

The new PnL is given by:

$$PnL = \max(0, K_p - S_T) - P(t, T, K_p) + C(t, T, K_{c1}) - \max(0, S_T - K_{c1}) - C(t, T, K_{c2}) + \max(0, S_T - K_{c2})$$

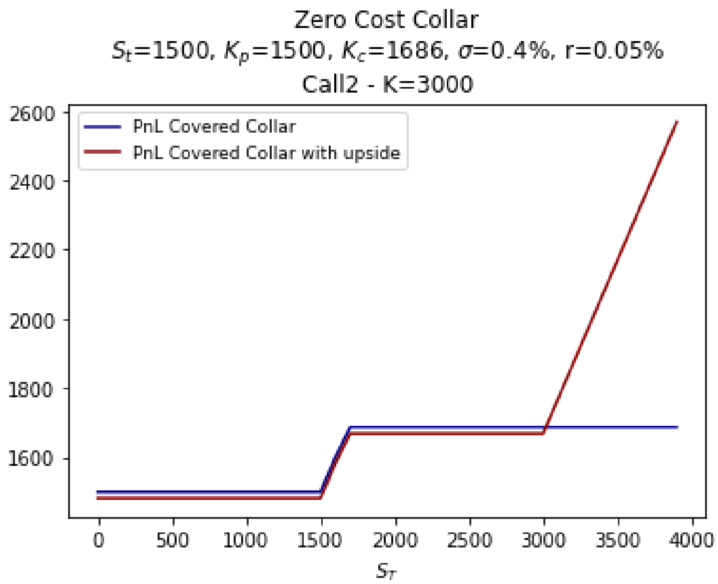
where:

- K_{c1} is the strike of the sold call in the collar
- K_{c2} is the strike of the DOTM call you bought

This can be simplified:

$$PnL = \max(0, K_p - S_T) - \max(0, S_T - K_{c1}) - C(t, T, K_{c2}) + \max(0, S_T - K_{c2})$$

Let's compare the PnL of the covered collar with and without the additional Call:



Zero Cost Collar
S_t=1500, K_p=1500, K_c=1686, σ=0.4%, r=0.05%
Call2 - K=3000
PnL Covered Collar
PnL Covered Collar with upside
2600
2400
2200
2000
1800
1600
0
500
1000
1500
2000
2500
3000
3500
4000
S_T